4
Hydrostatics

4.1 Pressure and Thrust

Since a liquid possesses weight, it exerts force on all bodies in contact with it. The ratio between the small force δF and the area δA on which it acts gives the pressure. Thus, pressure $= \delta F/\delta A$.

$$\text{Pressure at a point} = \underset{\delta A \to 0}{\text{Limit}} \left(\frac{\text{Force}}{\text{Area}}\right) = \underset{\delta A \to 0}{\text{Limit}} \left(\frac{\delta F}{\delta A}\right) = \frac{dF}{dA}.$$

Unit of pressure is Nm^{-2}. It can be shown that the hydrostatic pressure due to a liquid column of density ρ at a depth h from the surface $= h\rho g$.

The total force exerted by a liquid column on the whole of the area in contact with it is called thrust. Thus, *thrust = pressure × area.* Unit : Newton (*N*). The thrust is always normal to the plane area.

4.2 Thrust on a plane surface immersed in a liquid at rest.

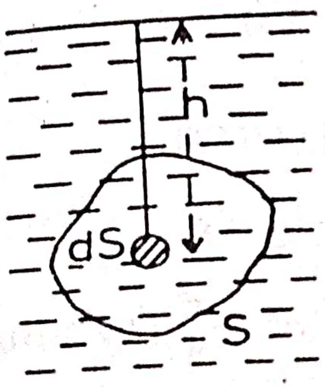

Fig. 4.1

Consider a plane lamina of area S immersed in a liquid of density ρ (Fig. 4.1). Divide the area into a very large number of small elements. Let dS be an elementary area at a depth h below the free surface of the liquid.

Thrust on dS at right angles to it $= h\rho g\, dS$.

$\therefore$ the resultant thrust on the whole surface

$$= \int h\rho g\, dS = \rho g \int h\, dS.$$

Let $\bar{h}$ be the depth of the C.G., from the liquid surface. From the theorem of moments,

$$h_1\, dS_1 + h_2\, dS_2 + h_3\, dS_3 + \ldots\ldots = \bar{h}S$$

$$\text{or} \int h\, dS = \bar{h}S$$

$\therefore$ resultant thrust on the immersed plane surface $= \bar{h}\rho g \times S$

= pressure at C.G. of area × area of the plane.

Example 1 : *A rectangular lamina (sides a,b) is immersed in water with its plane vertical and a side on the free surface. Show how to divide it into two parts by a horizontal line so that the thrusts on the two parts may be equal.*

Sol. Let $ABCD$ be a rectangular lamina of length a and breadth b immersed vertically in water with its plane vertical with the edge AB in the surface of water [Fig. 4.2]. Let the horizontal line EF at a depth x below the surface divide the area into two parts so that the thrusts on them are equal.

Let the horizontal line $B'C'$ divide the area into two parts, so that the thrusts on these portions are equal. Let this line be at a distance x above the vertex A.

Thrust on $AB'C'$

$$= \text{pressure at its C.G.} \times \text{area} = \left(h - \frac{2}{3}x\right) \rho g \times \frac{1}{2}x.B'C'$$

$$= \left(h - \frac{2}{3}x\right) \rho g \times \frac{1}{2}x \cdot \frac{ax}{h} \qquad \left(\because B'C' = \frac{ax}{h}\right)$$

Thrust on $AB'C' = \frac{1}{2} \times$ Thrust on the whole triangle.

$$\therefore \qquad \left(h - \frac{2}{3}x\right) \rho g \times \frac{1}{2}x \cdot \frac{ax}{h} = \frac{1}{2} \times \frac{1}{6} h^2 a\rho g$$

or $$\left(h - \frac{2}{3}x\right) x^2 = \frac{1}{6} h^3$$

or $$4x^3 - 6x^2h + h^3 = 0$$

or $$(2x - h)(2x^2 - 2xh - h^2) = 0$$

$$\therefore \qquad 2x - h = 0 \quad \text{or} \quad x = \frac{h}{2}.$$

4.3 Centre of pressure

We know that the liquid pressure acts *normally* at every point of the immersed area. The force acting on an elementary area like dS is $h\rho g dS$. The thrusts on different elements of the plane form a set of *like parallel forces*. All these parallel forces can be compounded into a resultant acting at some definite point on the plane area. This point is called the centré of pressure.

The centre of pressure of a plane surface in contact with a fluid is the point on the surface through which the line of action of the resultant of the thrusts on the various elements of the area passes.

Determination of Centre of pressure—General case -

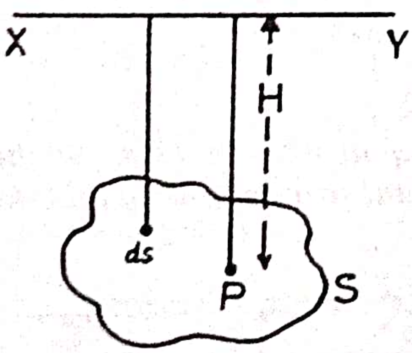

Fig. 4.7

Consider a plane surface of area S immersed vertically in a liquid of density ρ. Let XY be the surface of the liquid (Fig. 4.7).

Thrust on an elementary area dS at a depth $h = h\rho g dS$

Moment of this thrust about XY

$$= (h\rho g dS) \times h = h^2 \rho g dS$$

Resultant moment of all thrusts $= \int h^2 \rho g dS$

where the integration is carried over all the elements of the plane area.

Resultant thrust on the plane area $= \int h\rho g dS$

Let the centre of pressure of the plane area be at the point P. Let the distance of P from XY be H.

Moment of the resultant thrust about $XY = H\int h\rho g\, dS$.

By definition of the resultant of several forces, we get

Moment of resultant force = resultant of the moments of the forces.

$$H\int h\rho g\, dS = \int h^2 \rho g\, dS$$

or

$$H = \frac{\int h^2\, dS}{\int h\, dS}$$

or

The result holds good for any inclined position of the plane also.

4.4 Centre of pressure of a rectangular lamina immersed vertically in a liquid with one edge in the surface of the liquid.

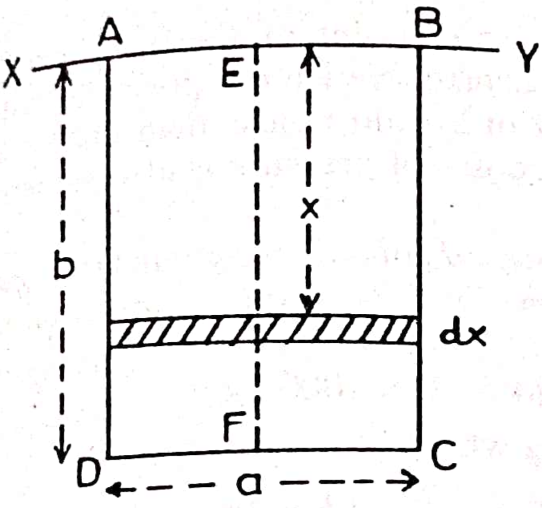

Fig. 4.8

Let $ABCD$ be a plane rectangular lamina immersed vertically in a liquid of density ρ with one edge AB in the surface XY of the liquid (Fig. 4.8). Let $AB = a$ and $AD = b$. Divide the rectangle into a number of narrow strips parallel to AB. Consider one such strip of width dx at a depth x below the surface of the liquid.

The thrust acting on the strip

$$= (x\rho g) \times (a dx) = x\rho g a\, dx$$

Moment of this thrust about AB

$$= (x\rho g a\, dx) \times x = x^2 \rho g a\, dx$$

Sum of the moments of the thrusts on all the strips $= \int_0^b x^2 \rho g a\, dx$

Resultant of the thrusts on all the strips $= \int_0^b x\rho g a\, dx$

Moment of the resultant thrust about $AB = H\int_0^b x\rho g a\, dx$

where H = depth of the centre of pressure below AB.

$$H\int_0^b x\rho g a\, dx = \int_0^b x^2 \rho g a\, dx$$

or $$H\rho g a\frac{b^2}{2} = \rho g a\frac{b^3}{3} \quad \text{or} \quad H = \frac{2}{3}b.$$

The thrust on every elementary strip acts through its midpoint. Hence the centre of pressure will lie on EF where E and F are the mid-points of AB and DC.

4.5. Centre of pressure of a triangular lamina immersed vertically in a liquid with its vertex in the surface of the liquid and its base horizontal.

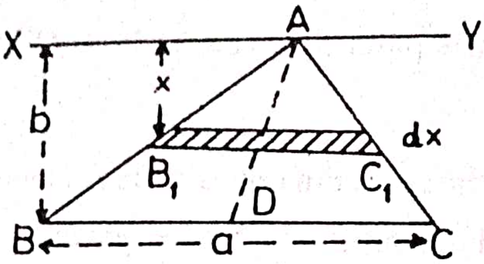

Fig.4.12

Sol. Let ABC be a triangular lamina immersed vertically in a liquid with its vertex A in the surface XY of the liquid and with its base BC horizontal (Fig 4.12). $BC = a$. Let the depth of the base of the lamina be b from the free surface of the liquid. Divide the triangle into a number of elementary strips of width dx parallel to the base BC. Consider one such strip $B_1 C_1$ of width dx at a depth x below the surface XY.

Area of the strip $B_1 C_1 = B_1 C_1\, dx = (ax/b)dx$

Thrust on the strip $B_1 C_1 = (x \rho g) \times (ax/b)\, dx$

Moment of this thrust about $XY = \left(\frac{ax^3 \rho g}{b}\, dx\right)$

Total moment due to all the strips $= \int_0^b \frac{a\rho g}{b} x^3\, dx.$

Resultant of the thrusts on all the strips $= \int_0^b \frac{a\rho g}{b} x^2 dx.$

Moment of the resultant thrust about $XY = H \int_0^b \frac{a\rho g}{b} x^2 dx.$

Here H= the depth of the centre of pressure below XY.
Since the two moments are equal,

$$\int_0^b \frac{a\rho g}{b} x^3\, dx = H \int_0^b \frac{a\rho g}{b} x^2\, dx.$$

or
$$\frac{a\rho g}{b}\left(\frac{b^4}{4}\right) = H \frac{a\rho g}{b}\left(\frac{b^3}{3}\right).$$

or
$$H = \frac{3}{4} b.$$

The centre of pressure lies on the line joining the mid-points of the strips. i.e., lies on the median AD at a depth $3b/4$ below the surface XY.

4.6 Centre of pressure of a triangular lamina immersed in a liquid with one side in the surface, when there is no external pressure.

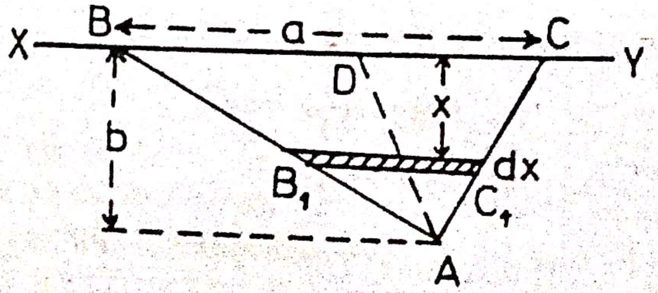

Fig. 4.13

5
Hydrodynamics

5.1 Equation of continuity

The equation of continuity is an expression of the law of conservation of mass in fluid mechanics. Fig. 5.1 represents a tube of varying cross-section through which a non-viscous incompressible fluid of density ρ flows. Let a_1 and a_2 be the cross- sectional areas of the tube at the points A and B. Let the velocity of the fluid at A and B be v_1 and v_2 respectively. Since the fluid is incompressible, in the steady state, mass of fluid entering the tube per second through the section A = mass of fluid leaving the tube per second through the section B.

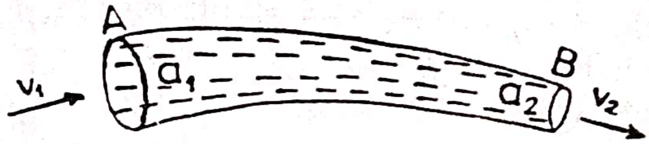

Fig. 5.1

Mass of fluid entering the tube per second across the section A

$$= a_1 v_1 \rho$$

Mass of fluid leaving the tube per second across the section B $= a_2 v_2 \rho$

$$\therefore \quad a_1 v_1 \rho = a_2 v_2 \rho \quad \text{or} \quad a_1 v_1 = a_2 v_2.$$

Thus the product av is constant along any given flow tube. It follows that the speed of flow through a tube is inversely proportional to the cross-sectional area of the tube. It means that where the area of cross-section of the tube is large, the velocity is small and *vice versa*.

Example. *Water flowing with a velocity of 3 m/s in a 4 cm diameter pipe enters a narrow pipe having a diameter of only 2 cm. Calculate the velocity in the narrow pipe.*

Here, $a_1 = \pi\,(0{\cdot}02)^2$; $v_1 = 3\ m/s$; $a_2 = \pi\,(0{\cdot}01)^2$; $v_2 = ?$

$$v_2 = a_1 v_1 / a_2 = \pi\,(0{\cdot}02)^2 \times 3 / \{\pi\,(0{\cdot}01)^2\} = 12\ m/s.$$

5.2. Energy of the liquid

A liquid in motion possesses three types of energy, viz., (*i*) potential energy (*ii*) kinetic energy and (*iii*) pressure energy.

(*i*) Potential energy. If we have a mass m of the liquid at a height h above the earth's surface, its P.E. $= mgh$.

P. E. per unit mass of the liquid $= gh$.

P. E. per unit volume of the liquid $= \rho gh$

(*ii*) Kinetic energy. The K. E. is the energy possessed by the liquid by virtue of its motion. The K. E. of a mass m of a liquid flowing with a velocity v is $\frac{1}{2}mv^2$.

$$\text{K. E. per unit mass of the liquid} = \frac{1}{2}v^2$$

$$\text{K. E. per unit volume of the liquid} = \frac{1}{2}\rho v^2$$

(*iii*) Pressure energy.

Consider an incompressible non-viscous liquid contained in a tank (Fig. 5.2). The tank has a side tube at an axial depth h below the free surface of the liquid in the tank. The side tube is fitted with a smooth piston. Let a be the area of cross-section of the side tube. Let ρ be the density of the liquid. Hydrostatic pressure of the liquid on the piston $= p = h\rho g$. Force on the piston $= pa$.

If the piston is moved inwards through a distance x, the work done $= pax$. This will force a mass ρax of the liquid into the tank.

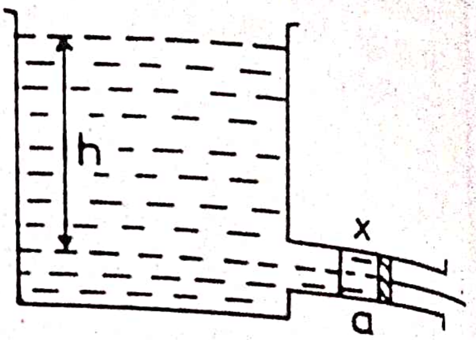

Fig. 5.2

This work done on the mass ρax accounts for an expenditure of energy pax. This is stored up as pressure energy of the same mass without imparting any velocity to it.

$$\text{Pressure energy per unit mass} = \frac{pax}{\rho\, ax} = \frac{p}{\rho}$$

Pressure energy per unit volume of the liquid $= p$

The three forms of energy possessed by a liquid under flow are mutually convertible.

5.3. Euler's Equation for Unidirectionl flow

Consider a very small element AB of a flowing liquid (Fig. 5.3). ρ is the density of the liquid. dA is the area of cross-section of the element. ds is the length of the element.

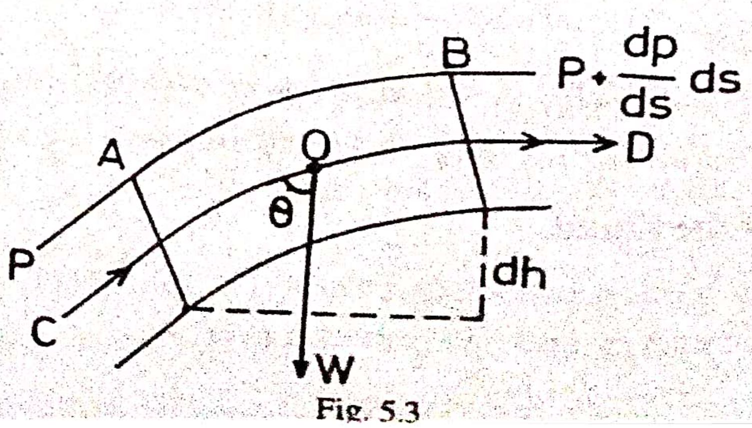

Fig. 5.3

Volume of the element $= dA.ds$
Mass of the element $= \rho\ dA.ds.$
Weight of the element $= \rho dA.ds.g.$
Let the streamline CD make an angle θ with the vertical at O.
Component of the weight of the element along the streamline

$$= \rho dA.ds.g.\cos\theta.$$

Let the pressure of the liquid at A be P.

Then, the pressure at $B = \left(P + \frac{dP}{ds}ds\right)$

Resultant force on the element $= PdA - \left(P + \frac{dP}{ds}ds\right)dA = -\frac{dP}{ds}ds\ dA.$

This force is directed from B to A. The component of the weight of the element is also directed from B to A.

The net force acting on the element $= -\frac{dP}{ds}ds\ dA - \rho\ ds\ dA\ g\cos\theta$

Let dh be the difference in heights between A and B. Then $\cos\theta = dh/ds$.

$\therefore$ the resultant force on the element is

$$F = -\frac{dP}{ds}ds\ dA - \rho\ ds\ dA\ g\frac{dh}{ds} = -\frac{dP}{ds}ds\ dA - \rho\ dA\ dh\ g$$

Let the velocity of the element of liquid be v. Then its acceleration is

$$a = \frac{dv}{dt} = \frac{ds}{dt}\frac{dv}{ds} = v\frac{dv}{ds}$$

According to Newton's II law of motion, $F = ma$.

$$-\frac{dP}{ds}ds\ dA - \rho\ dA\ dh\ g = \rho\ dA\ ds\ v\frac{dv}{ds}$$

or $$dP + \rho g\ dh = -\rho\ v\ dv$$

or $$\frac{dP}{\rho g} + dh + \frac{v\ dv}{g} = 0$$

This equation is called Euler's equation of motion.

Integrating, $$\frac{P}{\rho g} + \frac{v^2}{2g} + h = constant.$$

or $$\frac{P}{\rho} + \frac{1}{2}v^2 + gh = constant.$$

This is Bernoulli's equation.

5.4. Bernoulli's theorem

Statement. *The total energy of an incompressible non-viscous fluid flowing from one point to another, without any friction remains constant throughout the motion.*

Explanation. According to this theorem, the sum of kinetic, potential and pressure energies of any element of an incompressible fluid in streamline flow remains constant. Suppose the height of an element of fluid of density ρ above ground level is h. Let it be moving with a velocity v. Let it have a pressure p. Then, its total energy per unit volume is

$$E = \rho v^2/2 + \rho gh + p.$$

Bernoulli's theorem states that E is a constant.

If at two points in the fluid the velocities are v_1, v_2 the heights are h_1, h_2 and the pressures are p_1, p_2, then,

$$\rho v_1{}^2/2 + \rho gh_1 + p_1 = \rho v_2{}^2/2 + \rho gh_2 + p_2.$$

The K. E. per unit weight is called *velocity head* and is equal to $v^2/2g$. The P. E. per unit weight is called the *gravitational head* and is equal to h. The pressure energy per unit weight is called the *pressure head* and is equal to $p/\rho g$. Bernoulli's equation can be written as

$$v^2/(2g) + h + p/(\rho g) = constant$$

i. e., velocity head + gravitational head + pressure head = constant.

In the case of liquid flowing *along a horizontal pipe,* the gravitational head h is a constant.

$$\therefore \qquad \frac{v^2}{2g} + \frac{p}{\rho g} = \text{constant or } \frac{v^2}{2} + \frac{p}{\rho} = \text{constant}$$

or $$p + \rho v^2/2 = \text{constant}$$

or static pressure + dynamic pressure = constant .

This expression shows that greater velocity corresponds to a decrease in pressure and vice versa. *i. e.*, points of maximum pressure correspond to those of minimum velocity and vice versa. This principle may be used to determine fluid speeds by means of pressure measurements.

Example. Venturimeter, Pitot tube, etc.

Proof. Consider a fluid in stream line motion along a nonuniform tube (Fig. 5.4). Let A and B be two transverse sections of the tube at heights h_1 and

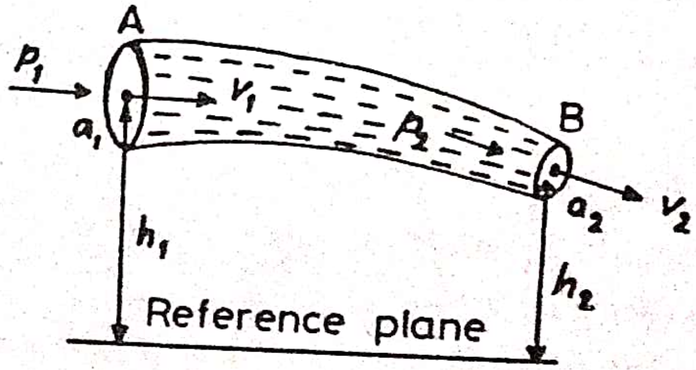

Fig. 5.4

h_2 from a reference plane (the surface of the earth). Let a_1 and a_2 be the areas of cross-section at A and B. Let v_1 and v_2 be the velocities of the fluid at A and

B. Let p_1 be the pressure at A due to the driving pressure head. Let p_2 be the pressure at B. Since $a_1 > a_2$, $v_2 > v_1$. Hence the fluid is accelerated as it flows from A to B.

Work done per second on the liquid entering at A is

$W_1 =$ Force at $A \times$ Distance moved by the liquid in 1 second

$$= p_1 a_1 \times v_1 = p_1 a_1 v_1$$

Work done per second by the liquid leaving the tube at B is

$$W_2 = p_2 a_2 v_2$$

$\therefore$ Net work done by the fluid in passing from A to B

$$W = W_1 - W_2 = p_1 a_1 v_1 - p_2 a_2 v_2$$

But

$$a_2 v_2 = a_1 v_1$$

$\therefore$

$$W = (p_1 - p_2)\, a_1 v_1$$

The work done on the liquid is used in changing its potential and kinetic energies.

$$\text{Decrease in P. E.} = (a_1 v_1 \rho)\, g(h_1 - h_2)$$

$$\text{Increase in K. E.} = \frac{1}{2}(a_1 v_1 \rho)\,(v_2{}^2 - v_1{}^2)$$

Hence, the total gain in the energy of the system when the liquid flows from A to B

$$= \frac{1}{2}(a_1 v_1 \rho)\,(v_2{}^2 - v_1{}^2) - (a_1 v_1 \rho g)\,(h_1 - h_2)$$

$$\therefore (p_1 - p_2)\, a_1 v_1 = \frac{1}{2}(a_1 v_1 \rho)\,(v_2{}^2 - v_1{}^2) - (a_1 v_1 \rho g)\,(h_1 - h_2)$$

or

$$p_1 + \frac{1}{2}\rho v_1{}^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2{}^2 + \rho g h_2$$

or

$$p + \frac{1}{2}\rho v^2 + h\,\rho\, g = \text{constant}$$

or

$$\frac{p}{\rho} + \frac{1}{2}v^2 + hg = \text{constant}$$

or

$$\frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{constant.}$$

$p/(\rho g)$ is called the pressure head. $v^2/(2g)$ is called the velocity head and h is called the gravitational head. Hence, Bernoulli's theorem may be stated as follows. *When an incompressible non-viscous fluid flows in stream line motion, the sum of the pressure head, velocity head and gravitational head remains constant throughout its motion.*

Example 1. *Water flows through a horizontal tube of varying section. At a place the velocity is* 0·3 *m/s when the pressure is* 0·016 m *of mercury. Find the pressure when the velocity is* 0·6 *m/s.*

The escaping liquid will strike the horizontal plane through the bottom of the vessel at a distance H, where $H = v \times t$

$$H = \sqrt{(2gh)} \times \sqrt{2h_1/g} = 2\sqrt{hh_1}$$

The range H is maximum for a given height $h + h_1$ if $h = h_1$.

Vena contracta. It is to be noted that the volume of the liquid that moves across the orifice in one second cannot be computed by multiplying the area of the orifice with the velocity of efflux. The streamlines converge as they approach the orifice. Therefore, the fluid velocities, as the jet leaves the hole, are not parallel to one another but have components inwards towards the centre of the stream. This inward momentum of the emergent fluid causes a contraction of the jet. After the jet has gone a little way, the contraction stops and the velocities become parallel. This point where the contraction of the jet stops and the fluid velocities become parallel is known as *vena contracta*. It is at this point that the velocity multiplied by area gives the rate of flow of the liquid. For an orifice of circular shape, the area of the *vena contracta* is about 65% of the area of the orifice.

Example 1. *A tank containing water has an orifice on one vertical side. If the centre of the orifice is 4·9 m below the surface level in the tank, find the velocity of discharge, assuming that there is no wastage of energy.*

$$v = \sqrt{2gh} = \sqrt{2 \times 9 \cdot 8 \times 4 \cdot 9} = 9 \cdot 8 \text{ ms}^{-1}.$$

Example 2. *Calculate the velocity of efflux of kerosene oil from a tank in which the pressure is 35150 kg wt per* m^2 *above the atmospheric pressure. The density of kerosene is 800 kg* m^{-3}.

Sol. Now,
$$h = \frac{P}{\rho} = \frac{35150}{800} = 43 \cdot 94 \text{ m}.$$

$$\therefore \quad v = \sqrt{2gh} = \sqrt{2 \times 9 \cdot 8 \times 43 \cdot 94} = 29 \cdot 35 \text{ ms}^{-1}.$$

(*ii*) **Venturimeter.** It is a device based on Bernoulli's principle. It is used for measuring the rate of flow of liquids in pipes. It consists of two wide conical tubes C_1 and C_2 with a constriction T between them. T is called the *throat*. Let the area of cross-section of C_1 and C_2 be A. Let a be the area of cross-section of the throat [Fig. 5.6].

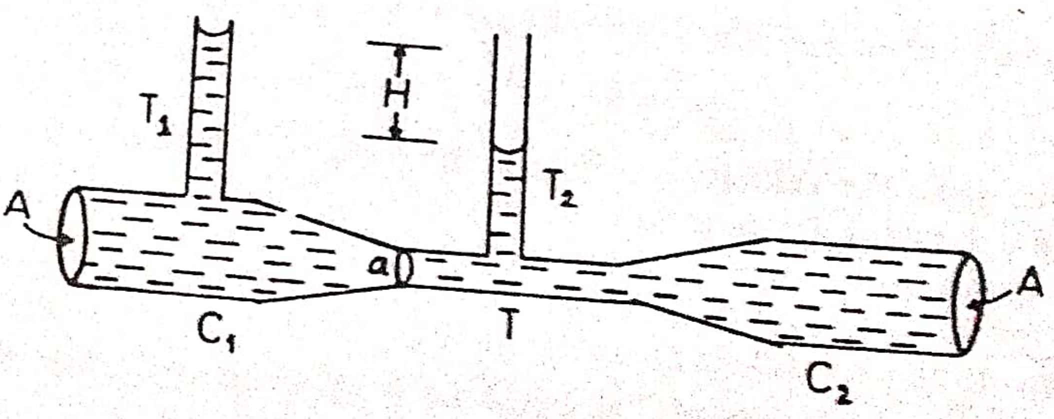

Fig. 5.6

When the flow is steady, let V be the volume of water flowing per second through the venturimeter.

Then, $V = Av_1 = av_2$.

Here v_1 = velocity in C_1 or C_2 and v_2 = velocity in T.

$$\therefore \quad v_1 = V/A \text{ and } v_2 = V/a.$$

Hence velocity of water in T is greater than the velocity in C_1 and C_2. Consequently, the pressure in T is smaller than the pressure in C_1 and C_2. This difference in pressure H is measured by the difference of the water levels in the vertical glass tubes T_1 and T_2 connected to C_1 and T respectively. Let p_1 and p_2 be the pressures in the wider limb and throat respectively.

According to Bernoulli's equation for a horizontal flow,

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

or

$$\frac{p_1 - p_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$$

The difference in pressure in C_1 and $T = p_1 - p_2 = H\rho g$.

Hence

$$\frac{H\rho g}{\rho g} = \frac{1}{2g}\left[\frac{V^2}{a^2} - \frac{V^2}{A^2}\right] = \frac{V^2}{2g}\left[\frac{A^2 - a^2}{A^2 a^2}\right]$$

$$\therefore \quad V = Aa\sqrt{\frac{2gH}{(A^2 - a^2)}}$$

The rate of flow of water through the pipeline can be determined by measuring H, and knowing the constants A, a, and g.

Example 1. *The diameter of the throat of a venturimeter is* 0·06 *m. When it is inserted in a horizontal pipe line of diameter* 0·1 *m, the pressure difference between the pipe and the throat equals* 0·08 *m of water. Calculate the rate of flow.*

Sol. Here $a = \pi\,(0{\cdot}03)^2 = 2{\cdot}826 \times 10^{-3}\ m^2$; $A = \pi\,(0{\cdot}05)^2$

$= 7{\cdot}85 \times 10^{-3}\ m^2$. $H = 0{\cdot}08\ m$. $V = ?$

$$V = Aa\sqrt{\frac{2gH}{A^2 - a^2}}$$

$$= (7{\cdot}85 \times 10^{-3})\,(2{\cdot}826 \times 10^{-3})\sqrt{\frac{2 \times 9{\cdot}8 \times 0{\cdot}08}{(7{\cdot}85 \times 10^{-3})^2 - (2{\cdot}826 \times 10^{-3})^2}}$$

$= 3{\cdot}793 \times 10^{-3}\ \text{m}^3/\text{s}$.

Example 2. *The diameter of a horizontal water pipe line at two points are* 0·05 *m and* 0·08 *m respectively. Calculate the difference in pressure between the two points if the rate of discharge of water is* $1{\cdot}887 \times 10^{-3}\ \text{m}^3/\text{s}$.

Sol. Let v_1 and v_2 be the velocities of flow at the two points. Let a_1 and a_2 be the areas of cross-section at the two points. Then,

$$a_1 v_1 = a_2 v_2 = \text{Rate of discharge of water.}$$

$$\therefore \quad \pi (0{\cdot}025)^2 v_1 = \pi (0{\cdot}04)^2 v_2 = 1{\cdot}887 \times 10^{-3}.$$

Hence, $v_1 = \dfrac{1{\cdot}887 \times 10^{-3}}{\pi (0{\cdot}025)^2} = 0{\cdot}9615 \text{ m/s}$

$$v_2 = \frac{1{\cdot}887 \times 10^{-3}}{\pi (0{\cdot}04)^2} = 0{\cdot}3756 \text{ m/s}.$$

Applying Bernoulli's theorem

$$\frac{p_1}{\rho} + \frac{1}{2} v_1^{\,2} = \frac{p_2}{\rho} + \frac{1}{2} v_2^{\,2}$$

or

$$p_2 - p_1 = \frac{\rho}{2} (v_1^{\,2} - v_2^{\,2})$$

$$p_2 - p_1 = \frac{1000}{2} [(0{\cdot}9615)^2 - (0{\cdot}3756)^2] = 391{\cdot}75 \text{ N/m}^2$$

Let H be the pressure difference in terms of metres of water.

$$H = \frac{p_2 - p_1}{\rho g} = \frac{391{\cdot}75}{1000 \times 9{\cdot}8} = 0{\cdot}04 \text{ m}.$$

Therefore, the pressure difference between the two points is 0·04 metres of water.

Example 3. *A venturimeter has a pipe diameter of 0·2 m and a throat diameter 0·15 m. The levels of water column in the two limbs differ by 0·1 m. Calculate the amount of water discharged through the pipe in one hour. Density of water = 1000 kg m^{-3}.*

Sol.
$$A = \frac{\pi d_1^{\,2}}{4} = \frac{\pi (0{\cdot}2)^2}{4} = 0{\cdot}0314 \text{ m}^2$$

$$a = \frac{\pi d_2^2}{4} = \frac{\pi (0{\cdot}15)^2}{4} = 0{\cdot}0177 \text{ m}^2$$

$$H = 0{\cdot}1 \ m.$$

$$\therefore \ V = Aa\sqrt{\frac{2gH}{A^2 - a^2}} = 0{\cdot}0314 \times 0{\cdot}0177 \sqrt{\frac{2 \times 9{\cdot}8 \times 0{\cdot}1}{(0{\cdot}0314)^2 - (0{\cdot}0177)^2}}$$
$$= 0{\cdot}03 \text{ m}^3 \text{ s}^{-1}.$$

Volume of water flowing per hour $= 0{\cdot}03 \times 3600 \text{ m}^3 hr^{-1} = 108 \text{m}^3/hr.$

(*iii*) **Pitot tube .** It is an instrument used to measure the rate of flow of water through a pipe-line. It is based on Bernoulli's principle. It consists of two vertical tubes *PQ* and *RS* with small apertures at their lower ends (Fig. 5.7). The plane of aperture of the tube *PQ* is parallel to the direction of flow of water. The plane of aperture of the tube *RS* faces the flow of water perpendicularly. The rise of the water column in the tube *RS* therefore, measures the pressure at *S*.

Let p_1 and p_2 be the pressures of water at Q and S respectively. Let v be the velocity of water at Q. Since the water is stopped in the plane of the aperture S of the tube RS, its velocity at S becomes zero. Hence the pressure increases to p_2 at S. Let H be difference of level in the two tubes. Applying Bernoulli's theorem to the ends Q and S,

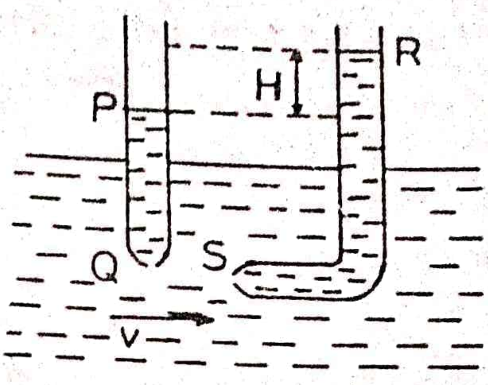

Fig. 5.7

$$\therefore \quad \frac{1}{2}v^2 + \frac{p_1}{\rho} = \frac{p_2}{\rho} \text{ or } v^2 = \frac{2}{\rho}(p_2 - p_1) = \frac{2}{\rho}H\rho g$$

$$\therefore \quad v = \sqrt{2gH}\,.$$

Rate of flow of water $= av$

where $a =$ area of cross-section of the pipe.

Example. *A Pitot tube is fixed in a main of diameter* 0·15 *m and the difference of pressure indicated by the gauge is* 0·04 m *of water column. Find the volume of water passing through the main in a minute.*

Here, $v = \sqrt{2gH} = \sqrt{2 \times 9{\cdot}8 \times 0{\cdot}04} = 0{\cdot}8854$ m/s

$\therefore$ volume of water flowing per second across the section

$= \text{area of section} \times \text{velocity} = \pi\,(0{\cdot}075)^2 \times 0{\cdot}8854.$

Volume of water passing through the main in a minute

$= [\pi\,(0{\cdot}075)^2 \times 0{\cdot}8854] \times 60 = 0{\cdot}9383 \text{ m}^3.$

Wings of an aeroplane: Wings of an aeroplane are shaped as shown in Fig. 5.8, with the lower surface being flat and the upper surface being curved. The figure shows a cross-section of the wing perpendicular to its length. A stream of air flowing past the wing divides into two branches, one above and one below the wing. The stream lines far away from the wing are straight. The stream-lines just above the wing get distorted due to the curvature of the upper surface of the wing. A tube of flow just above the wing will therefore get constricted. Its area of cross- section decreases. According to the equation of continuity ($a_1v_1 = a_2v_2$), the velocity of air increases. According to Bernoulli's theorem, this causes a decrease in pressure above the wing. The tubes of flow below the wing are not affected much. The pressure below the wing continues to be atmospheric. Therefore, there is a

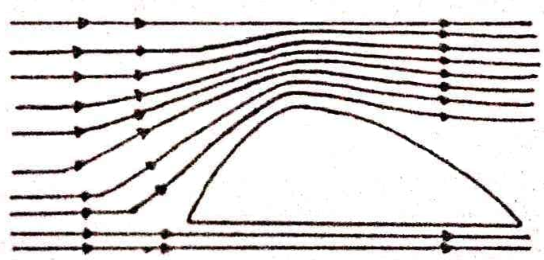

Fig. 5.8